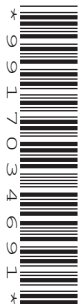


**Thursday 18 May 2023 – Afternoon**

**AS Level Mathematics B (MEI)**

**H630/01 Pure Mathematics and Mechanics**

**Time allowed: 1 hour 30 minutes**



**You must have:**

- the Printed Answer Booklet
- a scientific or graphical calculator



**INSTRUCTIONS**

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . When a numerical value is needed use  $g = 9.8$  unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**

- The total mark for this paper is **70**.
- The marks for each question are shown in brackets [ ].
- This document has **8** pages.

**ADVICE**

- Read each question carefully before you start your answer.

## Formulae AS Level Mathematics B (MEI) (H630)

**Binomial series**

$$(a+b)^n = a^n + {}^n C_1 a^{n-1} b + {}^n C_2 a^{n-2} b^2 + \dots + {}^n C_r a^{n-r} b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^n C_r = {}_n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!} x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

**Differentiation from first principles**

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

**Sample variance**

$$s^2 = \frac{1}{n-1} S_{xx} \quad \text{where } S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation,  $s = \sqrt{\text{variance}}$

**The binomial distribution**

If  $X \sim B(n, p)$  then  $P(X = r) = {}^n C_r p^r q^{n-r}$  where  $q = 1 - p$

Mean of  $X$  is  $np$

**Kinematics**

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2} at^2$$

$$s = \frac{1}{2}(u+v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2} at^2$$

- 1 A particle moves along a straight line. Its velocity  $v \text{ m s}^{-1}$  at time  $t \text{ s}$  is given by  $v = 2t + 0.6t^2$ .

Find an expression for the acceleration of the particle at time  $t$ . [2]

- 2 The height of the first part of a rollercoaster track is  $h \text{ m}$  at a horizontal distance of  $x \text{ m}$  from the start. A student models this using the equation  $h = 17 + 15 \cos 6x$ , for  $0 \leq x \leq 40$ , using the values of  $h$  given when their calculator is set to work in degrees.

(a) Find the height that the student's model predicts when the horizontal distance from the start is 40 m. [1]

(b) The student argues that the model predicts that the rollercoaster track will achieve a maximum height of 32 m more than once because the cosine function is periodic.

Comment on the validity of the student's argument. [2]

- 3 The points A and B have position vectors  $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$  and  $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$  respectively. The vector  $\overrightarrow{AC}$  is  $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$ .

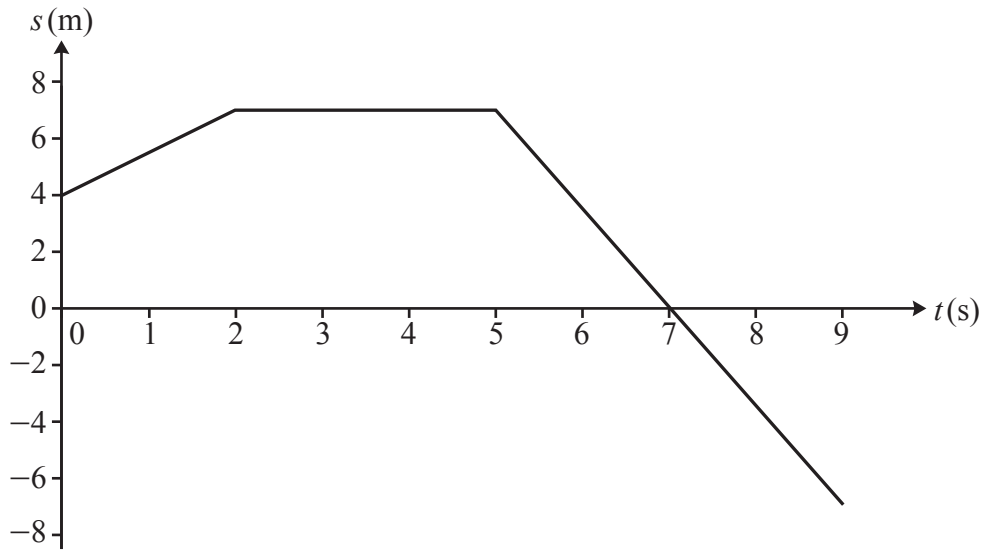
(a) Write down the position vector of C as a column vector. [1]

(b) Show that B is equidistant from A and C. [3]

- 4 **In this question you must show detailed reasoning.**

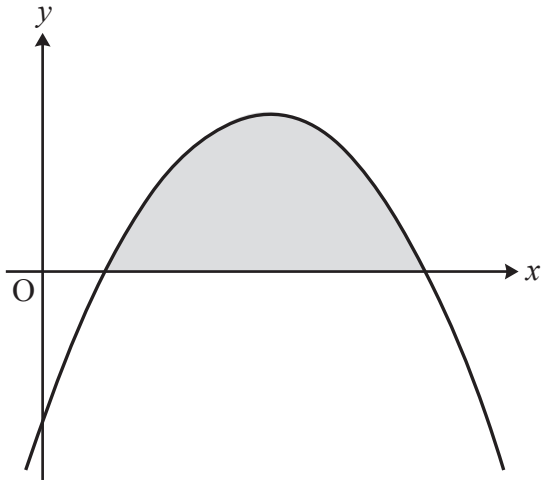
Solve the equation  $6 \cos^2 x + \sin x = 5$ , giving all the roots in the interval  $-180^\circ \leq x \leq 180^\circ$ . [5]

- 5 The graph shows displacement  $s$  m against time  $t$  s for a model of the motion of a bead moving along a straight wire. The points  $(0, 4)$ ,  $(2, 7)$ ,  $(5, 7)$  and  $(9, -7)$  are the endpoints of the line segments.



- (a) Find an expression for the displacement of the bead for  $0 \leq t \leq 2$ . [2]
- (b) Sketch the velocity-time graph for this model. [2]
- (c) Explain why the model may not be suitable at  $t = 2$  and  $t = 5$ . [1]
- 6 Show that the expression  $3x^3 + x^2 - 6x - 5$  can be written in the form  $(x + 2)(ax^2 + bx + c) + d$  where  $a$ ,  $b$ ,  $c$  and  $d$  are constants to be determined. [5]

7 In this question you must show detailed reasoning.



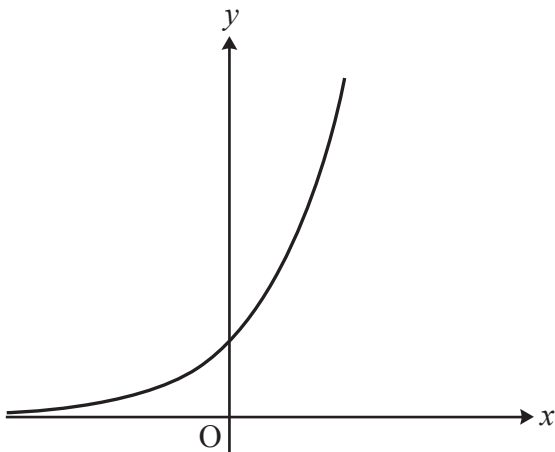
Find the exact area of the shaded region shown in the diagram, enclosed by the  $x$ -axis and the curve  $y = -3x^2 + 7x - 2$ . [6]

8 In this question you must show detailed reasoning.

(a) Find the centre and radius of the circle with equation  $x^2 + y^2 - 2x + 4y - 20 = 0$ . [4]

(b) Find the points of intersection of the circle with the line  $x + 3y - 10 = 0$ . [5]

- 9 The graph shows the function  $y = e^{2x}$ .



- (a) Describe the transformation of the graph of  $y = e^x$  that gives the graph of  $y = e^{2x}$ . [2]

A second function is defined by  $y = k + e^x$ .

- (b) A copy of the graph of  $y = e^{2x}$  is given in the Printed Answer Booklet.

Add a sketch of the graph of  $y = k + e^x$  in a case where  $k$  is a positive constant. [2]

- (c) Show that the two graphs do not intersect for values of  $k$  less than  $-\frac{1}{4}$ . [3]

- (d) In the case where  $k = 2$ , show that the only point of intersection occurs when  $x = \ln 2$ . [2]

- 10 Layla invests money in the bank and receives compound interest. The amount  $\pounds L$  that she has after  $t$  years is given by the equation  $L = 2800 \times 1.023^t$ .

- (a) (i) State the amount she invests. [1]

- (ii) State the annual rate of interest. [1]

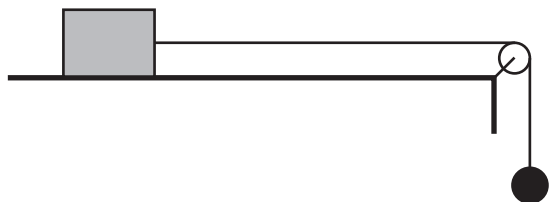
Amit invests  $\pounds 3000$  and receives 2% compound interest per year. The amount  $\pounds A$  that he has after  $t$  years is given by the equation  $A = ab^t$ .

- (b) Determine the values of the constants  $a$  and  $b$ . [2]

- (c) Layla and Amit invest their money in the bank at the same time.

Determine the value of  $t$  for which Layla and Amit have equal amounts in the bank. Give your answer correct to 1 decimal place. [3]

- 11 A block of mass 3 kg is at rest on a smooth horizontal table. It is attached to a light inextensible string which passes over a smooth pulley. This part of the string is horizontal. A sphere of mass 1.2 kg is attached to the other end of the string. The sphere hangs with this part of the string vertical as shown in the diagram. A horizontal force of magnitude  $F$  N is applied to the block to prevent motion.



- (a) Complete the copy of the diagram in the Printed Answer Booklet to show all the forces acting on the block and the sphere. [2]
- (b) Find the value of  $F$ . [2]

The force  $F$  N is removed, and the system begins to move.

- (c) The equation of motion of the block is  $T = 3a$ , where  $T$  N is the tension in the string and  $a$   $\text{ms}^{-2}$  is the acceleration of the block.

Write down the equation of motion of the sphere. [1]

- (d) Find the value of  $T$ . [2]

- 12 Points A, B and C lie in a straight line in that order on horizontal ground. A box of mass 5 kg is pushed from A to C by a horizontal force of magnitude 8 N. The box is at rest at A and takes 3 seconds to reach B. The ground is smooth between A and B. Between B and C the ground is rough and the resistance to motion is 28 N. The box comes to rest at C.

Determine the distance AC. [8]

**END OF QUESTION PAPER**

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